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### Investigating several alternatives for estimating the compound lead time demand in an (s,Q) inventory model

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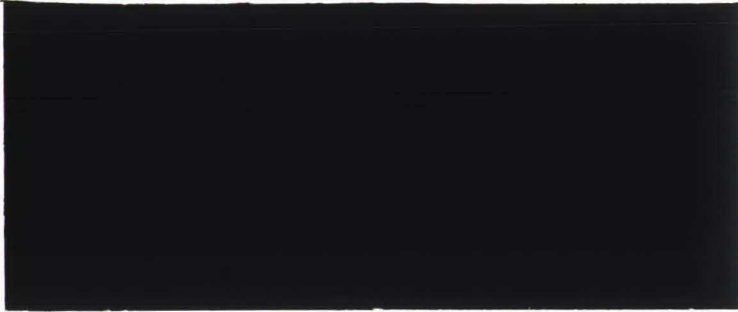
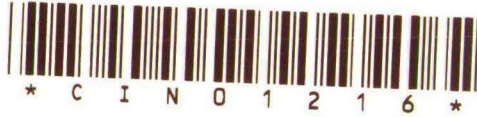
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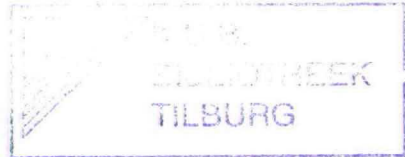
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INVESTIGATING SEVERAL ALTERNATIVES FOR  
ESTIMATING THE COMPOUND LEAD TIME  
DEMAND IN AN (s,Q) INVENTORY MODEL

L.W.G. Strijbosch and R.M.J. Heuts

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TITLE

INVESTIGATING SEVERAL ALTERNATIVES FOR ESTIMATING THE COMPOUND LEAD TIME  
DEMAND IN AN  $(s,Q)$  INVENTORY MODEL.

by

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ABSTRACT

Using Monte Carlo investigations this paper analyzes the cost differences between several alternative approximations for the lead time demand distribution (LTDD) in a continuous review  $(s,Q)$  inventory model. The information on LTDD is assumed to be build up by two components: demand per time unit and lead time. Enumeration methods, simulation and parametric approaches are used to obtain compound information on LTDD with help of the above individual components. Three important conclusions result:

- a) The simulation approach is simple and still able to take into account certain peculiarities in the lead time distribution in the most proper way.
- b) A lack of lead time information should be avoided as much as possible by a good information system. It is shown that extending the lead time information leads to drastical cost reductions in the inventory model.
- c) The gamma distribution appears to be a good approximation for the LTDD in many cases.

## 1. INTRODUCTION.

The approximation of the lead time demand distribution (LTDD) is a largely explored issue in the literature on inventory modelling (see e.g. [1] for an overview). LTDD can be considered a single random variable when the information process is such that it gathers information on demand during lead time directly. It can also be estimated by a combination of two variables, i.e. demand per time unit and lead time, or a combination of three variables: order intensity, order size, and lead time. Even when data are available, in most practical situations one is dependent on relatively small sets of empirical data to estimate the LTDD. The literature offers the practitioner not much guidance in choosing a general approach for a reliable estimation of the LTDD in case of certain peculiarities such as thick-tailness or bi-modality. Strijbosch and Heuts [2] consider the situation where empirical data is available as a sample of LTD-values and investigate parametric estimations of the LTDD versus a specific non-parametric estimation, the so-called kernel-density estimation. An extensive Monte-Carlo study indicated that always using a carefully constructed kernel-density approximation is a save strategy.

This paper studies several alternatives to approximate the LTDD based on empirical information on the demand per time unit and lead time. Let the lead time demand be given by

$$LTD = \sum_{i=1}^L D_i, \quad (1)$$

where  $L$  is the lead time in periods and  $D_i$  is the demand in period  $i$ .  $L$  is a positive discrete random variable and  $D_i$ ,  $i=1, \dots, L$  are independent identically distributed non-negative discrete random variables. Empirical information is supposed to be available as a sample of lead times  $l_1, \dots, l_t$  and a sample of demands per time unit  $d_1, \dots, d_k$ . The advantage of having an information system yielding empirical information on the lead time and the demand per time unit separately as compared with gathering information on demand during lead time directly, is that the LTDD can be approximated more precisely, since:

- Empirical information contained in the individual components is taken into account explicitly as advised by Bagchi et al. [1]. More detailed



information of underlying processes is used in this way, but information may also be lost when the compounding of the individual components has to be restricted to certain distributions for convolution reasons. This lack of information may be prevented by using suitable computer generating routines as indicated in the next sections.

- Certain peculiarities especially in the lead time (lead times exhibit significant variability in many cases, c.f. [3]), can be given the importance they need.

The paper proceeds as follows. The next section presents several estimation procedures for the LTDD. The third section provides information on several Monte Carlo investigations, whereas the conclusions are summarized in section four.

## 2. ESTIMATING THE LTDD.

### 2.1. The procedure of Lau and Zhao [4].

Lau and Zhao [4] (LZo for short) published an algorithm for the determination of the true LTDD when the distributions of L and D are given. Their procedure is based on an efficient enumeration of all possible demand combinations ('index combinations') for each possible lead time with corresponding probabilities, thus building the LTDD. It can also be used with empirical distributions for L and D. With increasing empirical information, the LTDD thus produced will approximate the true LTDD ever better. Consequently, it is useful to analyse the computational properties of this algorithm.

The practicability of the algorithm presented is mainly determined by the number of index combinations (NIC) involved. The required CPU-time is proportional with NIC. NIC is determined by the number of different periodic demands (not the values)  $n_D$  and the values of the possible lead times. Consider a situation where the lead times can vary from  $LT_1$  to  $LT_2$ . The number of different index combinations is given by

$$NIC(LT_1, LT_2, n_D) = \sum_{i=LT_1}^{LT_2} \binom{n_D + i - 1}{i}. \quad (2)$$

We used here the combinatorial property that there are  $\binom{n+r-1}{r}$  unordered samples of size r out of n with replacement. Properties are  $NIC(LT, LT, n_D)$

=  $NIC(n_D-1, n_D-1, LT+1)$ , and  $NIC(1, I, J) = NIC(1, J, I)$ . Table 1 presents some values of  $NIC(LT, LT, n_D)$  illustrating its explosive increase. Notice that  $NIC(10, 10, 10) = 92378$  and not 85268 as mentioned by LZo.

We have run the LZo procedure with  $LT_1=1$ ,  $LT_2=7, \dots, 11$ , and  $n_D=LT_2, \dots, 11$  leading to 15 observed CPU-times which turned out to be almost perfectly linear related with the number of index combinations. A linear regression (with intercept=0) of the required CPU-time on the number of index combinations revealed that each 10,000 index combinations costs approximately 2 seconds on a Vax-station 3100 (model 30). Thus the evaluation of the LTDD corresponding to the case  $LT_1=1$ ,  $LT_2=13$  and  $n_D=13$  (1,13,13) costs approximately 2000 seconds, or 33 minutes, and, for example, the cases (1,14,14) to (1,17,17) lead to CPU-times of 2.2, 8.6, 33.4 and 129.6 hours, respectively. LZo consider the very large case (1,50,50) and conclude that the memory requirement for such a case is only 2500, not realizing, probably, that a complete determination of the corresponding LTDD

Table 1. Values of  $NIC(LT, LT, n_D)$  for  $LT=1, \dots, 13$ , and  $n_D=2, \dots, 13$ .

LT	$n_D$											
	2	3	4	5	6	7	8	9	10	11	12	13
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	6	10	15	21	28	36	45	55	66	78	91
3	4	10	20	35	56	84	120	165	220	286	364	455
4	5	15	35	70	126	210	330	495	715	1001	1365	1820
5	6	21	56	126	252	462	792	1287	2002	3003	4368	6188
6	7	28	84	210	462	924	1716	3003	5005	8008	12376	18564
7	8	36	120	330	792	1716	3432	6435	11440	19448	31824	50388
8	9	45	165	495	1287	3003	6435	12870	24310	43758	75582	125970
9	10	55	220	715	2002	5005	11440	24310	48620	92378	167960	293930
10	11	66	286	1001	3003	8008	19448	43758	92378	184756	352716	646646
11	12	78	364	1365	4368	12376	31824	75582	167960	352716	705432	1352078
12	13	91	455	1820	6188	18564	50388	125970	293930	646646	1352078	2704156
13	14	105	560	2380	8568	27132	77520	203490	497420	1144066	2496144	5200300

would require somewhere between a billion and a trillion years of CPU-time on a VAX-station 3100, far most of the CPU-time spending to the determination of negligible contributions to the LTDD. This discussion makes clear that calculating the LTDD with the LZo procedure is infeasible for most combinations of LT and  $n_D$ . Since, however, the LTDD will only be used in the context of a certain inventory model, it is not necessary to perform such a calculation fully. Several studies indicate that using only part of the information contained in the empirical data sets in an inventory model with low stock-out risk can be satisfactory (c.f. [5]), that is, not necessarily leads to larger average total relevant costs. LZo suggest to reduce  $n_D$  to at most 10 frequency classes, and converting the lead time range (from days to weeks, e.g.) such that the maximum lead time is 10 periods.

An alternative approach would be a modification of the LZo procedure such that negligible contributions to the LTDD are skipped systematically. It turns out, however, that such a modification leads to a much more complicated algorithm, while underestimating the -most important- right tail of the LTDD.

## 2.2. A simulation approach.

A procedure which automatically attains the required effect of skipping negligible contributions to the LTDD is simulation. The less probable an index combination, the more likely that no time is spent to the corresponding contribution to the approximated LTDD. When using standard procedures (e.g. of the NAG-library) a program for the approximation of the LTDD by simulation can be very simple (see Appendix). A sample of size  $m$  is drawn with replacement from the (empirical) distribution for the lead time:

$l'_1, \dots, l'_m$ . Then, for  $i=1, \dots, m$ , a sample of size  $l'_i$  is drawn with replacement from the (empirical) distribution for the demand per time unit:  $d'_{i1}, \dots, d'_{il'_i}$ . Accumulating frequencies of  $\sum_{j=1}^{l'_i} d'_{ij}$ ,  $i=1, \dots, m$ , and dividing the frequencies by  $m$ , yields, already with relatively small values of  $m$ , a very close approximation  $LTDD_{sim}$  to the  $LTDD_{LZ}$  which can be obtained by employing LZo's procedure. Let, for example,  $Prob(L=5)=0.1$ , and  $Prob(D=10)=0.1$ . Then, for some  $i$ , the probability of selecting  $l'_i=5$



and  $d'_{ij}=10$ ,  $j=1,\dots,5$ , equals  $10^{-6}$  which is at the same time the corresponding contribution to  $\text{Prob}(\text{LTD}=50)$ . Some try-outs clarify that a few minutes of simulation suffice to produce an  $\text{LTDD}_{\text{sim}}$  showing a remarkable likeness with the  $\text{LTDD}_{\text{LZ}}$  based on the same data and obtained after 50 hours calculating on a VAX-station 3100. In other words, as  $m$  grows to infinity,  $\text{LTDD}_{\text{sim}}$  converges to the  $\text{LTDD}_{\text{LZ}}$  based on the same data, but the convergence rate is very high.

Section 3.1 describes a Monte Carlo investigation which compares LZo's procedure and the simulation approach in the context of an  $(s,Q)$  inventory model.

### 2.3. A parametric approach.

Still another alternative is the fit of a standard theoretical distribution e.g. based on estimated mean, variance, skewness and kurtosis. Several papers have been published with formulas for the first four moments of LTD, given the first four moments of L and D. Kottas and Lau [6,7] give incorrect third and fourth central moments of LTD. In [4] a reference is made to [8] where the correct third and fourth central moments can be found, but nothing is said, however, about errors in [6,7] as should have been done. Furthermore, the correct results of the first four central moments of LTD were already obtained by [9], c.f. [10]. Carlson [9] used cumulant generating functions, and from his results it is easy to derive the four central moments. An example of a distribution which is characterized by four parameters is the Schmeiser-Deutsch (SD) distribution (c.f. [11]). From empirical data these parameters can be estimated using the first four empirical moments. Several authors have studied the SD distribution in the context of inventory modelling (c.f. [2,5,12]). As the gamma distribution is widely used (c.f. [13,14,15,16], personal communications with Philips managers) for the approximation of the LTDD, we included in our study a strategy based on a gamma distributed LTDD with parameters determined from the empirical data. Note that all empirical information is reduced in this case to two figures: mean and variance.

Section 3.2 describes a Monte Carlo investigation which compares LZo's

procedure and the two parametric approaches mentioned above in the context of an  $(s, Q)$  inventory model.

### 3. MONTE CARLO INVESTIGATIONS.

Via Monte Carlo methods we are going to analyze the effect of using several alternative estimation procedures for LTD in an  $(s, Q)$  inventory model of an expected average costs per unit time minimization type (see [2,17] for a more detailed analysis), where  $s$  and  $Q$  are simultaneously optimized. Thus, the following model is used:

$$EAC(s, Q, F) = kM/Q + cM + h(Q/2 - M\lambda + s) + (hQ_{\max}/Q) \int_s^{\infty} (q-s) dF(q), \quad (3)$$

where  $EAC(s, Q, F)$  is the expected average costs per time unit given the decisions  $s$  and  $Q$  and  $F(q)$ , the cumulative distribution of demand during the lead time,  $M$  the expected demand per time unit,  $\lambda$  the expected lead time,  $h$  the inventory holding cost per unit per unit of time,  $\beta$  the backlogging cost per unit short just before a replenishment order arrives,  $k+cQ$  the ordering cost per order,  $Q_{\max} = M(h\lambda/2 + \beta)/h$ . In practice one has to work with a parametric or non-parametric estimation  $\hat{F}$  of  $F$  on the basis of empirical information. Minimizing  $EAC(s, Q, F)$  leads to the optimal values  $s^*$  and  $Q^*$ , while minimizing  $EAC(s, Q, \hat{F})$  leads to the estimations  $\hat{s}$  and  $\hat{Q}$ . As an inventory model with a cost criterion is used, this paper investigates the cost-effect  $EAC(\hat{s}, \hat{Q}, F) - EAC(s^*, Q^*, F)$  of using a particular estimator  $\hat{F}$  instead of the true  $F$ , which is unknown in practice. For detailed information on the determination of  $\hat{s}$  and  $\hat{Q}$  in various cases see [2]. As an exact determination of the LTDD  $F$  in the case of a compound LTD is mostly infeasible (c.f. section 2.1),  $F$  has been approximated very accurately by the method of simulation (c.f. section 2.2).

Before setting up a Monte Carlo study, we have to reflect on the situations to be simulated. Inspired by several real life data three theoretical distributions are constructed for the lead time as mentioned in Table 2. When only fast moving items are considered, it appears reasonable to assume a normal or, for example, a gamma distributed demand per time unit. Therefore we based the theoretical distribution for the demand per time unit on a sample with size 10,000 from a normal distribution

(ignoring negative sample values) with  $\mu=200, \sigma=70$  (A), or from a gamma distribution with  $\mu=200, \sigma=140$  (B). The six combinations A1,B1, A2,B2, A3 and B3 lead to 'true' LTDD's (LTDD<sub>A1</sub> to LTDD<sub>B3</sub>) which are approximated by the method of LTDD<sub>sim</sub> with very large  $m$  (e.g. 500,000). It would have been impossible of course to determine these LTDD's with the procedure of LZ0.

Table 2. Theoretical distributions for L.

lead time $l$	Prob(L=l)		
	1	2	3
1	0.23	0.05	-
2	0.29	0.10	-
3	0.16	0.30	-
4	0.09	0.10	-
5	0.07	0.05	-
6	0.03	0.03	-
7	0.04	0.07	-
8	0.04	0.20	0.50
9	0.03	0.07	0.50
10	0.02	0.03	-

Figure 1 displays these six LTDD's. The shape of the LTDD is determined both by a central limit theorem effect and the shape of the underlying distributions for L and D.

INSERT FIGURE 1 ABOUT HERE

### 3.1. Comparing LTDD<sub>LZ</sub> and LTDD<sub>sim</sub>, a comparison between non-parametric procedures.

The first part of this study is a Monte Carlo comparison between LTDD<sub>LZ</sub> and LTDD<sub>sim</sub>. Consider the situation where observed lead times vary from  $LT_1$  to  $LT_2$  and  $n_D$  different periodic demands  $d_1, \dots, d_{n_D}$  (in increasing

order) are registered with frequencies  $f_1, \dots, f_{n_D}$ . Often it will be necessary to reduce  $n_D$  to a smaller number  $\underline{n}_D$  for the determination of  $LTDD_{LZ}$ . Such a reduction can be performed in many ways. We chose for the next method. Write  $n_D = \underline{n}_D * a + b$ , where  $0 \leq b < n_D$ ,  $a > 0$ ,  $a, b$  integer. Then let

$$\left. \begin{aligned} \underline{f}_j &= \sum_i f_i \\ \underline{d}_j &= \sum_i d_i f_i / \underline{f}_j \end{aligned} \right\} \begin{aligned} j=1, \dots, b: & \text{ summation is from } (j-1)(a+1)+1 \text{ to } j(a+1); \\ j=b+1, \dots, \underline{n}_D: & \text{ summation is from } b(a+1)+(j-1-b)a+1 \\ & \text{to } b(a+1)+(j-b)a. \end{aligned} \quad (4)$$

The next numerical example with  $n_D=16$  and  $\underline{n}_D=7$  (so that  $a=b=2$ ) clarifies this procedure:

$$\begin{aligned} f_{1, \dots, 16} &= 4 \ 2 \ 2 \ 6 \ 5 \ 8 \ 4 \ 3 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ , \\ d_{1, \dots, 16} &= \underline{0 \ 1 \ 2} \ \underline{3 \ 4 \ 5} \ \underline{7 \ 10} \ \underline{15 \ 16} \ \underline{20 \ 30} \ \underline{60 \ 100} \ \underline{200 \ 400} \ , \\ \underline{f}_{1, \dots, 7} &= 8 \quad 19 \quad 7 \quad 3 \quad 2 \quad 2 \quad 2 \ , \\ \underline{d}_{1, \dots, 7} &= 3/4 \quad 78/19 \quad 58/7 \quad 46/3 \quad 25 \quad 80 \quad 300 \ . \end{aligned}$$

Note that classifying original observations histogram-like (according to a previously fixed classification) is not the same and would result in a larger loss of information in general. Several interesting issues can be formulated now:

- (i) What is the cost-effect of reducing  $n_D$  by the method described above;
- (ii) Which strategy reduces the costs more:
  - reducing  $n_D$  such that the determination of  $LTDD_{LZ}$  costs no more than  $u$  CPU-seconds, or
  - spending the same  $u$  CPU-seconds (controlled for by  $m$ ) to obtain  $LTDD_{sim}$ .

The Monte Carlo setup for the comparison of  $LTDD_{LZ}$  and  $LTDD_{sim}$  will be described now. For each of the six combinations the following is repeated  $T$  times. A sample of lead times  $l_1, \dots, l_t$  and a sample of demands per time unit  $d_1, \dots, d_k$ , where  $k = \sum_{i=1}^t l_i$ , is taken. Vary the required CPU-time  $u$  from 0.04 to 1 second (1 second turns out to be sufficiently large). Calculate for each pair of samples and each value of  $u$  both  $LTDD_{LZ}$  and  $LTDD_{sim}$ . Based on these approximations of the  $LTDD$  and on the 'true'  $LTDD_{Xi}$  ( $X=A, B$ ;



$i=1,2,3$ ) the mean relative bias (MRB) of the expected total relevant costs can be determined now for both the sim and the LZo approach. The MRB characteristic is defined as follows:

$$\text{MRB} = \frac{1}{T} \sum_{k=1}^T [\text{EAC}_k - \text{EAC}] / \text{EAC}, \quad (5)$$

where  $\text{EAC} = \text{EAC}(s^*, Q^*, F)$  is the true cost,  $k$  is the simulation step and  $\text{EAC}_k = \text{EAC}(\hat{s}, \hat{Q}, F)$  is the cost corresponding with the (generally not optimal) decisions  $\hat{s}, \hat{Q}$  based on the approximated LTDD  $\hat{F}$  in that step. MRB for both approaches is plotted against varying values of  $u$  and for three different values of  $t$ , viz. 5, 10, 20, in Figure 2.

INSERT FIGURE 2 ABOUT HERE

We only present the results for combination A1 as the results for the other combinations are comparable. The values for  $h$ ,  $k$  and  $c$  are 0.2, 50 and 0, respectively. The value for  $\beta$  is such that the true service level is approximately 90%. The values for  $t$  are taken small as lead time information in practice tends to be sparse.  $T$  is chosen as 500 for this Monte Carlo experiment. The results are partly surprising. Spending more CPU-time to the approximation of the LTDD leads to lower average total relevant costs (as expected), but the effect of more CPU-time is negligible after spending a few tenths of a second, which is (e.g. for  $t=20$ ) a very little fraction of the time required to obtain  $\text{LTDD}_{\text{LZ}}$  based on the same data (with large  $n_D$ ). A second result is that the difference between the approximations  $\text{LTDD}_{\text{LZ}}$  and  $\text{LTDD}_{\text{sim}}$  tends to disappear very fast for increasing computation time  $u$ . The higher values for small  $u$  obtained with  $\text{LTDD}_{\text{sim}}$  indicate that the simulation approach should not be used with small  $m$  (e.g.  $<500$ ;  $m=500$  roughly corresponds with 1/2 Cpu-second in our study). For too small values of  $m$  the effect of the simulation error is larger than the effect of the statistical error caused by the empirical distributions of  $L$  and  $D$ . A conclusion of this investigation is that the procedure of LZo is useful in practice but can be approximated very well

by the much simpler simulation approach. Further, lead time information appears to be very cost-effective. So, the information system in practice should be such, that enough lead time information is carefully collected and updated.

### 3.2. Comparing $LTDD_{LZ}$ , $LTDD_{\gamma}$ and $LTDD_{SD}$ , a comparison between a non-parametric and two parametric procedures.

The second part of this study is a Monte Carlo comparison between  $LTDD_{LZ}$ ,  $LTDD_{\gamma}$  and  $LTDD_{SD}$ . For the determination of  $LTDD_{LZ}$  in this case, using the results of section 3.1, we reduce in all cases  $n_D$  to a smaller number  $\underline{n}_D$  such that calculation time is less than 1 CPU-second. Thus we compare three strategies for handling the empirical data: LZo, a non-parametric way of estimating the LTDD (within 1 second calculation time),  $\gamma$ , fitting a gamma distribution using the estimated mean and variance of the LTD, and SD, fitting a Schmeiser-Deutsch distribution. Section 2.3 refers to the literature where formulas can be found for the estimation of the first four empirical moments of LTD given the first four empirical moments of L and D. In order to investigate the effect of various values of  $h$ ,  $\beta$  and  $k$ , we chose for  $h$  and  $k$  the combinations (0.1,10), (1,10), (0.1,500) and (1,500) while, again, the value for  $\beta$  is such that the true service level is approximately 90% and  $c$  is fixed at 0. The value for  $T$  is 200 for this Monte Carlo experiment. Analyzing Table 3, which reports the results, we may formulate the next findings. The best of the LZo and  $\gamma$  strategies is almost always better than the SD strategy. Obviously, the advantage of a possible better fit through the 4-parameter character of the SD distribution is destroyed by the typical properties of that distribution. Furthermore, it turns out that applying the promising procedure of LZo can enlarge the costs unnecessarily, especially when the sample size of the empirical distribution for the lead time is small, or when the LTDD can be very well approximated by a gamma distribution. Always using the  $\gamma$  strategy seems to be a save strategy, except for one situation: when  $t$  is not too small and the LTDD is far from gamma-like, then LZo's procedure can be advantageous (c.f. A2).

**Table 3. MRB values for various inventory model settings and three different approaches for the determination of the LTDD.**

LTDD	t	k	h=0.1				h=1			
			$\beta$	LZo	$\gamma$	SD	$\beta$	LZo	$\gamma$	SD
A1	5	10	3.4	0.78	0.49	0.91	30.9	1.30	0.43	2.31
		500	8.8	0.15	0.14	0.15	42.8	0.41	0.32	0.42
	10	10	3.4	0.34	0.24	0.41	30.9	0.40	0.18	0.92
		500	8.8	0.08	0.08	0.07	42.8	0.20	0.18	0.20
	20	10	3.4	0.09	0.07	0.10	30.9	0.12	0.08	0.30
		500	8.8	0.04	0.04	0.03	42.8	0.07	0.06	0.06
A2	5	10	2.0	0.37	0.38	0.43	15.3	0.57	0.42	0.92
		500	7.9	0.14	0.18	0.15	30.1	0.26	0.30	0.28
	10	10	2.0	0.10	0.17	0.12	15.3	0.11	0.17	0.19
		500	7.9	0.05	0.08	0.05	30.1	0.08	0.14	0.09
	20	10	2.0	0.04	0.13	0.06	15.3	0.05	0.13	0.10
		500	7.9	0.02	0.04	0.02	30.1	0.03	0.10	0.04
A3	5	10	1.3	0.08	0.07	0.07	12.8	0.11	0.09	0.14
		500	7.3	0.03	0.03	0.02	24.0	0.06	0.05	0.05
	10	10	1.3	0.04	0.04	0.04	12.8	0.06	0.05	0.10
		500	7.3	0.01	0.01	0.01	24.0	0.03	0.03	0.03
	20	10	1.3	0.02	0.02	0.02	12.8	0.03	0.02	0.07
		500	7.3	0.01	0.01	0.01	24.0	0.01	0.01	0.01
B1	5	10	4.1	0.65	0.45	0.83	40.9	1.17	0.56	2.75
		500	9.3	0.18	0.18	0.18	50.5	0.41	0.34	0.42
	10	10	4.1	0.20	0.16	0.27	40.9	0.27	0.18	0.77
		500	9.3	0.08	0.07	0.07	50.5	0.15	0.13	0.16
	20	10	4.1	0.09	0.07	0.12	40.9	0.11	0.08	0.27
		500	9.3	0.04	0.04	0.04	50.5	0.08	0.06	0.08
B2	5	10	3.3	0.28	0.26	0.37	29.6	0.34	0.28	0.90
		500	8.4	0.12	0.13	0.12	40.5	0.21	0.21	0.22
	10	10	3.3	0.11	0.12	0.14	29.6	0.12	0.12	0.28
		500	8.4	0.06	0.07	0.06	40.5	0.09	0.10	0.10
	20	10	3.3	0.05	0.07	0.07	29.6	0.05	0.07	0.16
		500	8.4	0.02	0.03	0.03	40.5	0.04	0.06	0.05
B3	5	10	2.5	0.18	0.15	0.17	21.8	0.20	0.16	0.34
		500	8.3	0.08	0.07	0.07	34.2	0.14	0.12	0.12
	10	10	2.5	0.10	0.08	0.09	21.8	0.11	0.08	0.16
		500	8.3	0.04	0.04	0.04	34.2	0.08	0.06	0.07
	20	10	2.5	0.04	0.03	0.06	21.8	0.05	0.04	0.10
		500	8.3	0.02	0.02	0.02	34.2	0.03	0.03	0.04



#### 4. CONCLUSION.

In this study it is assumed that empirical data are available on both the lead time in certain time units and on the demand per time unit. There are many situations where a gamma distribution is a save choice to approximate the LTDD based on the empirical data. When the empirical distribution for the lead time indicates that lead time is unimodal with a relatively large mean, the central limit theorem applies strongly and the LTDD can be reasonably fitted by a gamma distribution in general. One has to be careful, however, in using the gamma distribution in some cases. When the maximum lead time is relatively small, the shape of the LTDD is mainly determined by the shape of the distribution for  $D$ , which may be far from gamma-like. Further, peculiarities such as multimodality in the lead time distribution will be reflected in the LTDD. Using a fitted gamma distribution (or another parametric distribution) could have unwanted effects on the costs. In all such cases where it is reasonable to doubt on the Gauss or gamma-like shape of the LTDD, we recommend the LZo procedure (or, equivalently, the corresponding simulation approach). Furthermore, the Monte Carlo experiments indicate strongly that sample sizes for the lead time should not be smaller than 10. Otherwise costs can be easily more than 30% higher on the average as compared to the costs corresponding with the optimal choices for the inventory parameters  $s$  and  $Q$ . Spending a little more costs on improving lead time information will in general lead to large costs reductions in the inventory model. So, this is an investment which pays off.



## APPENDIX

An algorithm for the approximation of the 'true' LTDD via simulation based on the theoretical distributions for L and D, or on the corresponding empirical distributions can be very simple as the following algorithm illustrates:

Algorithm for the approximation of an LTDD by simulation:

```
k := 0;
repeat
  k := k+1;
  LT := {a drawing with replacement from the distribution of L};
  LTD := 0;
  for i := 1 to LT do
    begin
      d := {a drawing with replacement from the distribution of D}
      LTD := LTD+d;
    end;
  LTDD[LTD] := LTDD[LTD]+1/m;
until k=m;
```

The drawings with replacement can be done easily by using the NAG-procedures G05EXF and G05EYF, or other equivalent procedures.

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LEGENDS TO FIGURES

FIGURE 1.

Theoretical LTDD's A1,B1, A2,B2, A3 and B3 obtained by using the simulation approach with large value of  $m$ .

FIGURE 2.

Comparison of the MRB for both LZo's procedure (dotted line) and the simulation approach (solid line), three different sample values  $t$ , and varying values of the required CPU-time. The LTDD of combination A1 is used.

FIGURE 1; A1

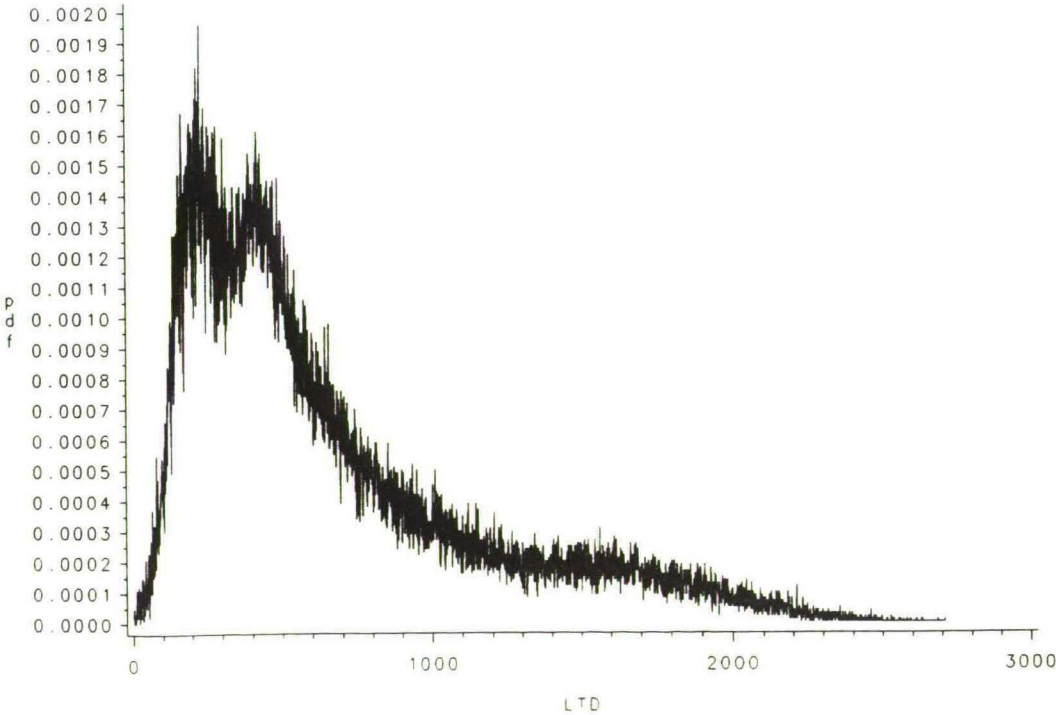




FIGURE 1; B1

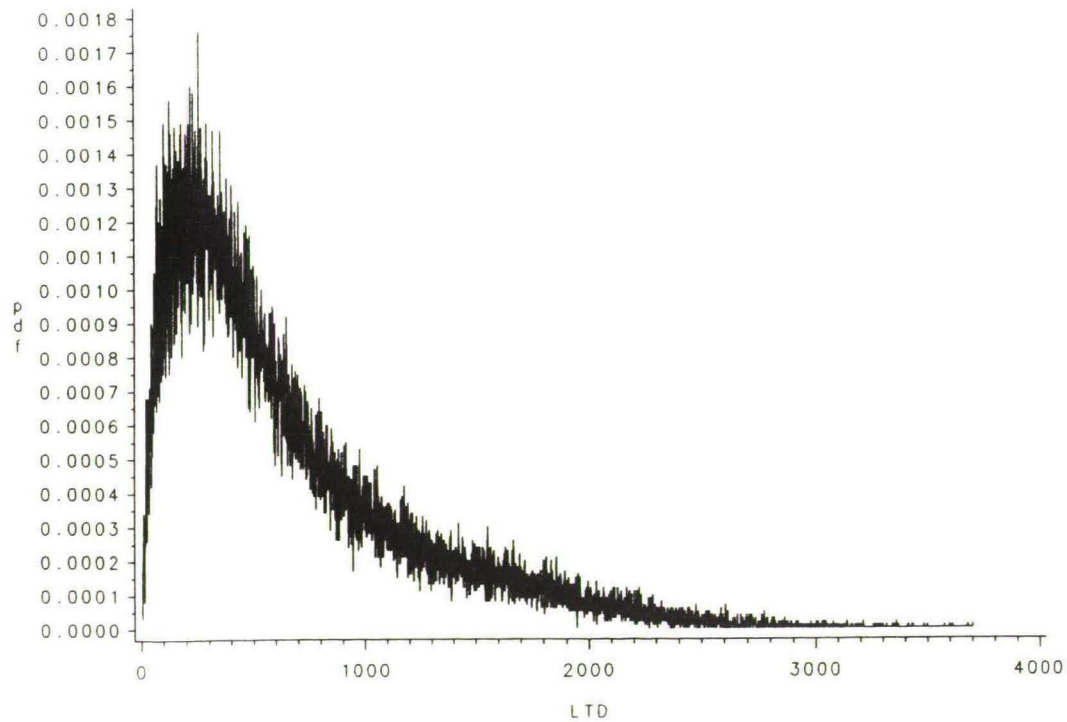


FIGURE 1; A2

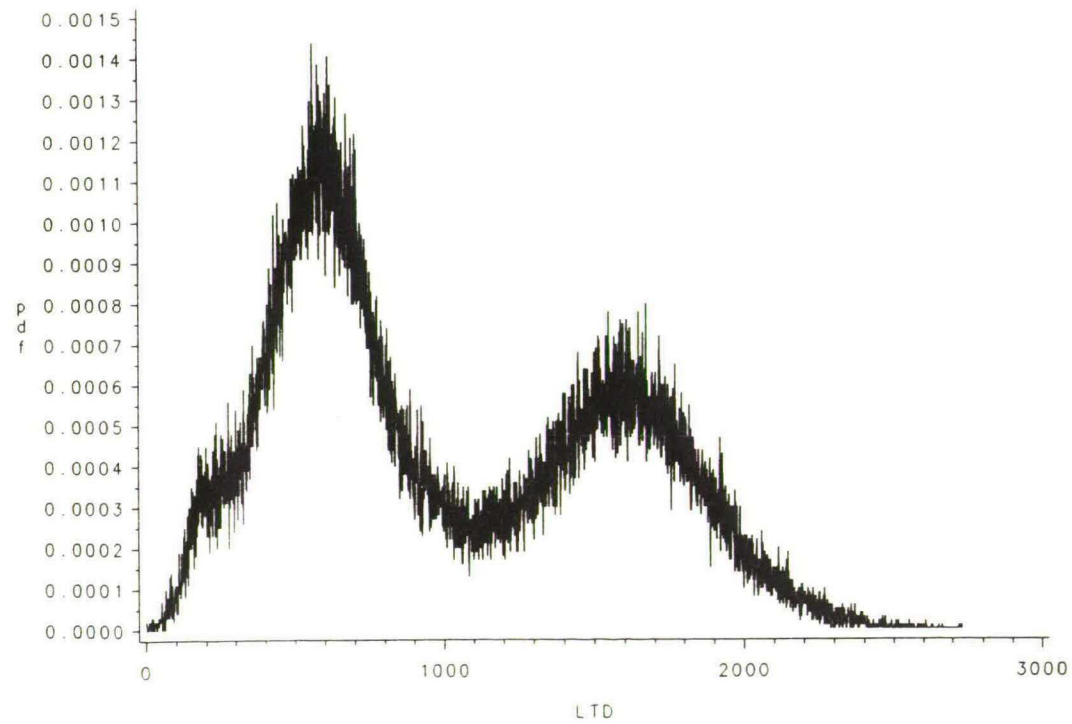


FIGURE 1; B2

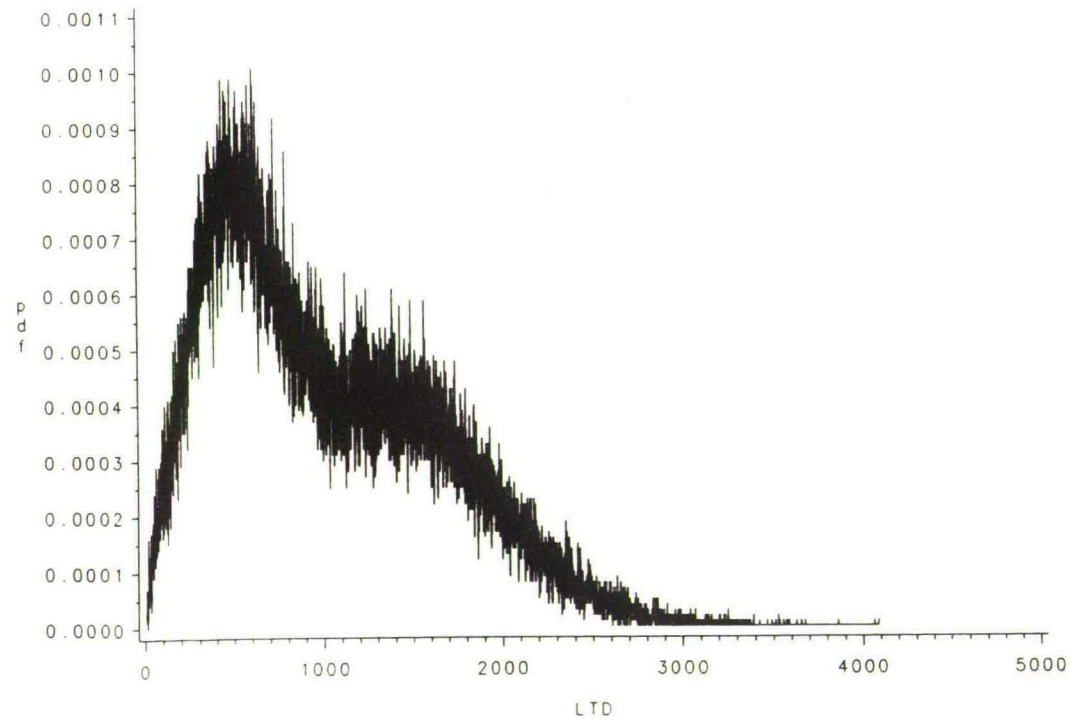


FIGURE 1; A3

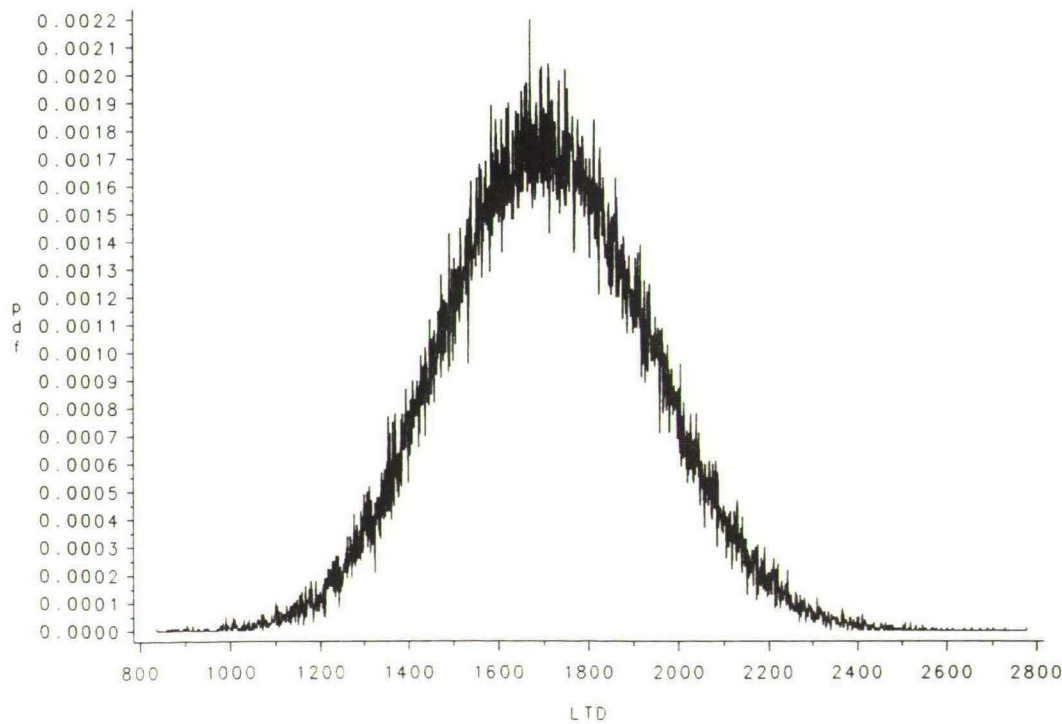




FIGURE 1: B3

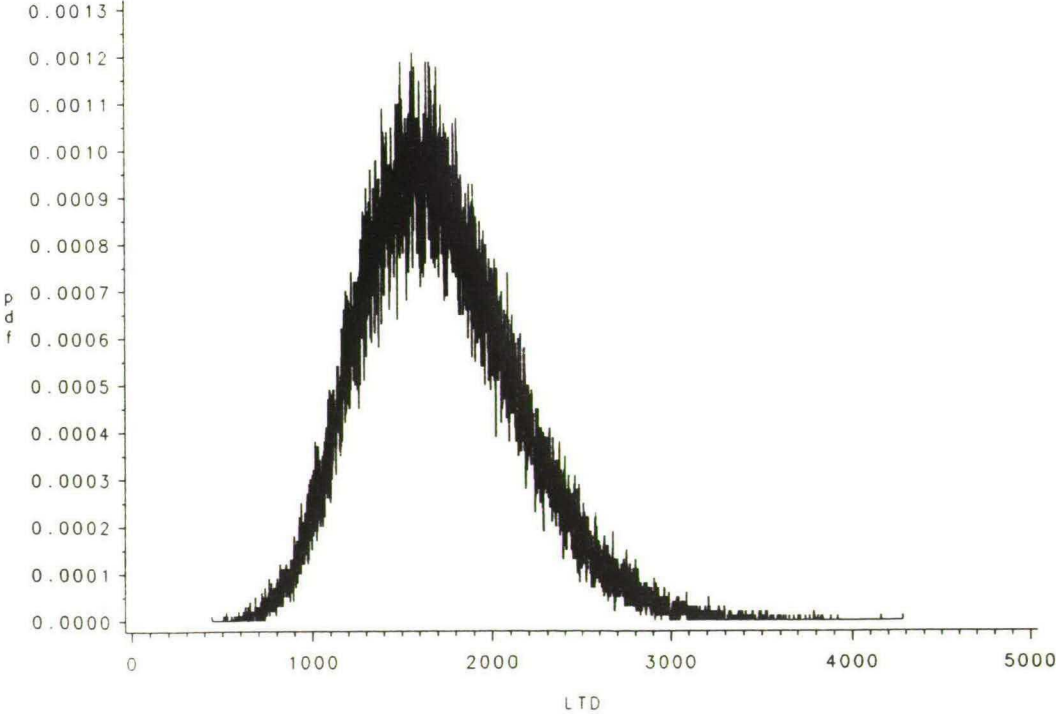
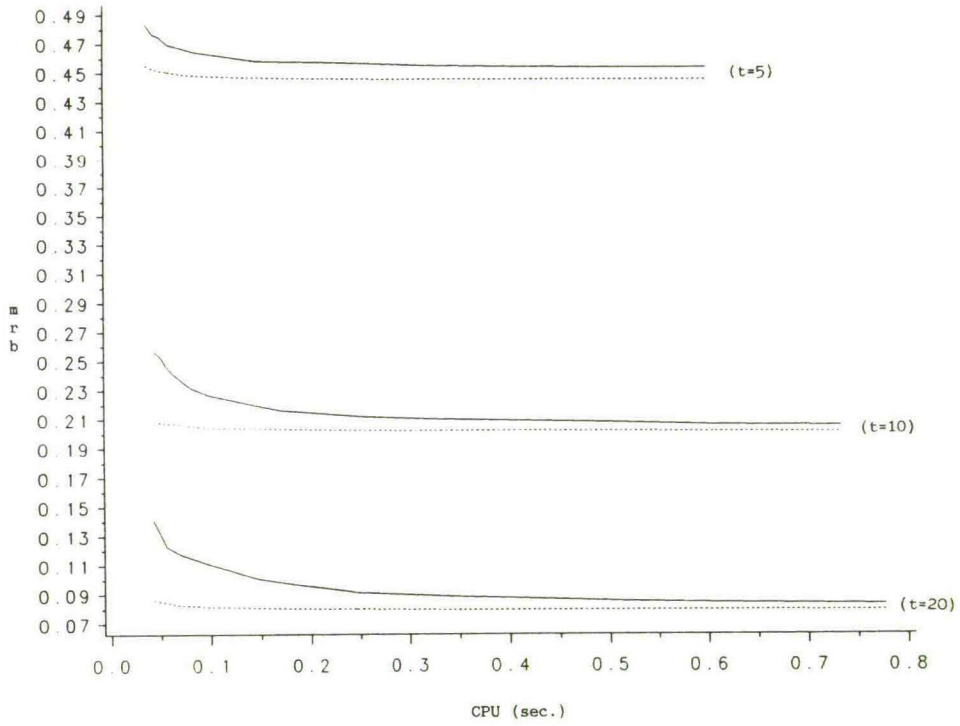


FIGURE 2



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